RADIANT HEAT TRANSFER IN HYDROGEN PLASMA

G. A. Koval'skaya, V. G. Sevast'yanenko, and R. I. Soloukhin

UDC 533.9:536.24:537.53

We describe the integral radiation characteristics integrated over the entire spectrum for the rapid calculation of heat transfer in systems containing hot hydrogen, with variable temperature and pressure fields.

In [1-3] we developed a new integral method for the calculation of heat transfer by radiation in the real spectrum when there is a line structure. Previously, before solving the gasdynamics problem, we carry out a complete integration with respect to frequency, and also with respect to angles if there is sufficient symmetry in the system. We calculate matrices of functionals of two types which depend only on the macroparameters of the problem — the temperatures, the pressures, and the geometric dimensions. The calculation of the heat exchange reduces to making a selection from the matrix of characteristics and integrating them once. The time required for calculating the radiation field becomes small in comparison with the time required to solve the gasdynamics problem, while in conventional methods the relation between the amounts of time spent is reversed. The essence of the method is that the absorptive capacity of the gas is independent of the specific distribution of the parameters along the path of absorption and is determined only by the integral of the absorption coefficient taken along this path. The real distributions of the parameters may be replaced by a set of model splines in such a way that the absorptive capacity remains approximately the same. The method enables us to work numerically on a gasdynamics network whose step depends only on the gradients of temperature, pressure, and velocity and does not require the introduction of special networks whose step depends on the optical density. It is fairly simple and accessible to a wide class of specialists in thermophysics and gasdynamics who have no special training in the field of atomic spectroscopy. The integral characteristics consist of a set of smooth functions that can be easily stored in computer memories.

The method of effective occupancies [3-6], based on a large amount of experimental material, enabled us to obtain such values of the spectral radiation characteristics as were suitable for measurement. It takes account of the effects of the gradual transition of the line spectrum to the continuous spectrum; for the contours of spectral lines which have no analytic expression, we used the results of exact calculations.

In the present study we describe the tables of the partial characteristics ΔI (or Som) and ΔSim, which will be used in calculating the flux fields and flux divergence fields in systems of any geometric form with arbitrary distribution of pressure and temperature.

The partial intensity ΔI has the form

$$\Delta I(T_\xi,\ P_\xi,\ T_X,\ P_X,\ x)=\int_0^\infty I_\nu^0(T_\xi)\,k'_\nu(T_\xi,\ P_\xi)\exp\left(-\int_0^x k'_\nu(\eta)\,d\eta\right)d\nu. \tag{1}$$

The source for calculating the divergence fields is

$$\mathrm{Som}(T_X,\ P_X,\ T_\xi,\ P_\xi,\ x)=\int_0^\infty I_\nu^0(T_X)\,k'_\nu(T_X,\ P_X)\exp\left(-\int_0^x k'_\nu(\eta)\,d\eta\right)d\nu. \tag{2}$$

The partial efflux is

$$\Delta\,\mathrm{Sim}(T_\xi,\ P_\xi,\ T_X,\ P_X,\ x)=\int_0^\infty [I_\nu^0(T_\xi)-I_\nu^0(T_X)]\,k'_\nu(T_\xi,\ P_\xi)\,k'_\nu(T_X,\ P_X)\exp\left(-\int_0^x k'_\nu(\eta)\,d\eta\right)d\nu. \tag{3}$$

Institute of Theoretical and Applied Mechanics, Siberian Branch, Academy of Sciences of the USSR Novosibirsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 6, pp. 977-983, December, 1983. Original article submitted June 14, 1982.

 0022-0841/83/4506-1424$07.50

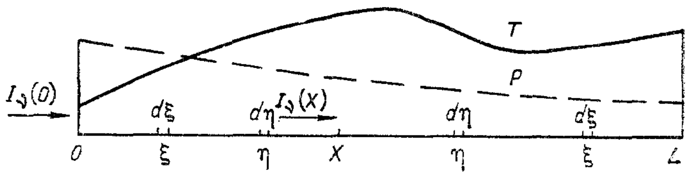

Fig. 1. Scheme showing the calculation of the integrals (1)-(3).

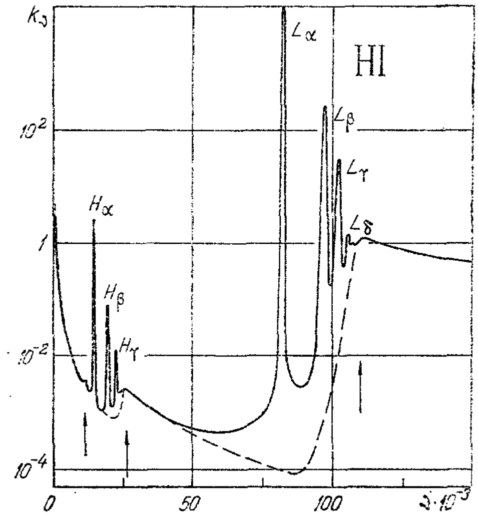

Fig. 2. Spectrum of the absorption coefficient k_ν of hydrogen for T = 15,000°K, P = 1 bar. ν, cm^{-1}.

Here T_ξ, P_ξ are the parameters of the source (Fig. 1); T_X, P_X are the parameters at the calculation point (or the sink); $x = |\xi - X|$ is the geometric distance from the point ξ of the source to the calculation point X. The inner integrals in (1)-(3) are taken along the path specified by the modeling splines, so that the coefficient of absorption $k'_\nu(\eta)$ in these integrals is determined by the distribution of the parameters T(η) and P(η):$k_\nu \equiv k_\nu(T_\eta, P_\eta)$.

It can be seen that the source Som (2), except for the difference in notation, coincides with the partial intensity ΔI(1), so that all we need calculate are two functionals out of the three.

The composition of the hydrogen plasma, taking into account the effects of the interaction in it, is calculated in [4, 5]. The optical spectrum of atomic hydrogen is simpler in structure than the spectra of complex atoms. Calculating this spectrum, however, is made difficult by the fact that there is no analytical description for the contours of the spectral lines, in the broadening of which the linear Stark effect plays an essential role. Because of this, in calculations we must use the relevant data in the form of tables. The theory of broadening of hydrogen lines, first developed by Grim et al., was later improved, and today there are relatively detailed tables of normalized contours [7], as well as tables obtained by using relaxation theory [8]. In the contours tabulated in [8], Doppler broadening has also been introduced with the aid of a convolution procedure.

A complete study of the spectrum of atomic hydrogen and a calculation of the energy radiated by an isothermal volume of such a plasma were first carried out in [9]. Detailed calculations of the optical characteristics of hydrogen plasma, taking account of the contribution made by molecular bands, were given in [10]. The tables obtained in that study are the most complete in the literature. In the present study, in constructing the spectrum for calculating the partial characteristics, we have taken into account all the most important radiation processes whose role is explained in [10]. The interaction of the particles in the plasma is taken into account according to the method of [3-6]. The intensities of the ionizing fields are given with the corresponding terms in square brackets in [3, 6]. In calculating the bound–free transitions from the ground and excited states, we took account both of the optical shift and of the reduction in the occupancy of those levels whose photo-

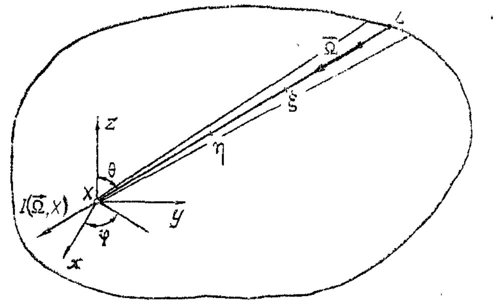

Fig. 3. Scheme for the three-dimensional integration of the radiation field.

ionization contributes to the given frequency. The photoionization cross section [11, 12] was extrapolated to the long-wave region in accordance with the principle of spectroscopic stability. The free–free transitions in the proton fields were calculated by Kramer's formulas [11]. The free–free transitions in the fields of negative H^- ions were taken into account in accordance with the data of [13], and in the fields of neutrals in accordance with those of [14]. In the latter case we used the approximations proposed in [15]. The photodetachment of electrons from negative ions was calculated by using the cross sections from [16, 17], an approximation to which was also given in [15]. The line spectrum was calculated according to the tables of [8], into which we introduced resonance broadening by using the convolution procedure. For weak lines we used the approximate expressions given in [18]. The occupancies of all the bound states were calculated by the method of effective occupancies. At low temperatures and high pressures, a considerable contribution to the absorption coefficient is made by the molecular bands. These processes were taken into account by means of the approximate expressions of [19, 20] with a smoothed rotational structure. In Fig. 2, as an example, we show the behavior of the coefficient of absorption of the hydrogen plasma as a function of frequency. The effect of the smooth transition from a line spectrum to a continuous spectrum can be clearly seen.

The error in the calculation of the integral characteristcs from the known coefficient of absorption is 1%. The main error is due to the coefficient of absorption, which is calculated with an error of 5-10% in different ranges.

The tables of partial characteristics calculated in the present study are designed for the calculation of the intensity, fields of fluxes, and divergences of fluxes of radiant energy in systems of any geometry with a characteristic dimension up to 1 m, containing hot hydrogen, with an arbitrary distribution of pressure in the interval from 0.1 to 30 bar and temperature from room temperature to 20,000°K.

The tables have five input parameters (T_ξ, P_ξ, T_X, P_X, x). The source temperature T_ξ is given from 8000 to 20,000°K with a step of ΔT_ξ = 2000°K. The contribution made to the heat transfer by the segments with temperature T_ξ < 8000°K is small and may be set equal to zero. The temperature T_X of the calculation point (or the sink) is given from 2000 to 20,000°K with a step of ΔT_X = 2000°K. For points of the volume at which the temperature T_X < 2000°K we can set the temperature equal to 2000°K and be sure that the error will be small.

For convenience of interpolation, the pressures and geometric dimensions are given on a logarithmic scale. The logarithm of the pressure (in bars) varies from −1 to +1.5, which ensures that we can calculate values from 0.1 to about 30 bar. The logarithm of x (in centimeters) varies from −2 to +2, which corresponds to dimensions from 0.1 mm to 1 m.

In the tables we give the mantissa and the order. Thus, the number 81 + 01 corresponds to a value of $0.81 \cdot 10^1$.

The dimensions of the partial characteristics and the results obtained by interpolating in the tables (measuring the step of the network in centimeters) are the following: ΔI — $W \cdot cm^{-3} \cdot sr^{-1}$; ΔSim — $W \cdot cm^{-4} \cdot sr^{-1}$; I — $W \cdot cm^{-2} \cdot sr^{-1}$; ∇I — $W \cdot cm^{-3} \cdot sr^{-1}$; S — W/cm^2; ∇S — W/cm^3.

For the calculation of the intensity at the point X of the system in the direction of $\vec{\Omega}$ (Fig. 3) we must calculate the integral

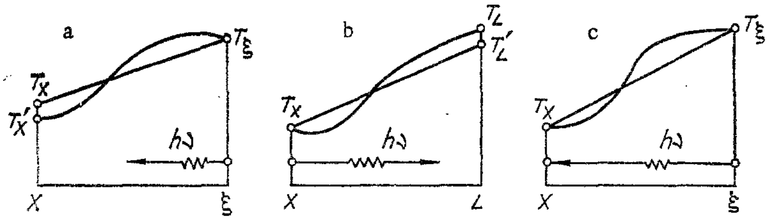

Fig. 4. Approximation of segments of working profiles by the modeling splines.

$$I(X,\ \Omega)=\int_X^L \Delta I(T_\xi,\ P_\xi,\ T_X,\ P_X,\ x)\,d\xi. \tag{4}$$

Here ΔI is the partial intensity, taken from the tables; T_ξ and P_ξ are the temperature and the pressure at the point ξ (the parameters of the source); T_X and P_X are the temperature and the pressure at the calculation point X. For a crude calculation the latter parameters are taken directly at the point X, while for more refined calculations they are computed by the formulas (Fig. 4a)

$$T_X'=\frac{2}{x}\int_X^\xi T(\eta)\,d\eta-T_\xi, \tag{5}$$

$$x=|\xi-X|.$$

$$P_X'=\frac{2}{x}\int_X^\xi P(\eta)\,d\eta-P_\xi, \tag{6}$$

The approximations (5) and (6) take account of the fact that the photons emitted at the point ξ and reaching the point X, the determining quantity is the optical density of the path $\xi\to X$. The step of the integration, d_ξ for (4) and d_η for (5) and (6), may be selected from the available gasdynamic network. There is no need to construct a special network for calculating the radiation field by the method of partial characteristics. The upper limit L in the integral (4) is determined by the boundary of the radiating volume (Fig. 3).

After calculating the intensity field, we can calculate the flux field by the formula

$$\mathbf{S}(X)=\int_{(4\pi)} I(X,\ \Omega)\,\Omega\, d\Omega. \tag{7}$$

When we use a spherical system of coordinates (Fig. 3), the components of the vector S(X) have the form

$$S_x(X)=\int_0^\pi d\theta\int_0^{2\pi} d\varphi I(X,\ \theta,\ \varphi)\sin^2\theta\cos\varphi, \tag{8}$$

$$S_y(X)=\int_0^\pi d\theta\int_0^{2\pi} d\varphi I(X,\ \theta,\ \varphi)\sin^2\theta\sin\varphi, \tag{9}$$

$$S_z(X)=\int_0^\pi d\theta\int_0^{2\pi} d\varphi I(X,\ \theta,\ \varphi)\sin\theta\cos\theta. \tag{10}$$

To determine the divergences of the fluxes of radiation, we must first calculate at the point X the quantity

$$\nabla I(X,\ \Omega)=\mathrm{Som}(X,\ L)-\int_X^L \Delta\,\mathrm{Sim}(T_\xi,\ P_\xi,\ T_X,\ P_X,\ x)\,d\xi. \tag{11}$$

Here Som(X, L) is the effective source of photons emitted from the point X in the direction opposite to Ω, taking account of the photon absorption along the path $X\to L$. The source Som(X, L) is chosen from the matrix ΔI according to the rule

$$\mathrm{Som}(X,\ L)=\Delta I(T_X,\ P_X,\ T_L,\ P_L,\ x). \tag{12}$$

Here T_X and P_X are source parameters, and therefore in the matrix $\Delta I(T_1,\ P_1,\ T_2,\ P_2,\ x)$ these are the first pair of parameters, i.e., they must be substituted into the matrix ΔI to replace the parameters T_ξ and P_ξ. The parameters T_L and P_L are the temperature and pressure of the boundary of the ray $X\to L$ (Fig. 1). In the exact formulation these parameters should, if possible, be calculated by formulas analogous to (5) and (6) (Fig. 4b):

$$T_L' = \frac{2}{x}\int_X^L T(\eta)\,d\eta - T_X, \tag{13}$$

$$P_L' = \frac{2}{x}\int_X^L P(\eta)\,d\eta - P_X. \tag{14}$$

T_L' and P_L' in formula (12) are the second pair of parameters of the matrix ΔI. In the expressions (13), (14), as well as in the matrix $\Delta I(T_1, P_1, T_2, P_2, x)$, when formula (12) is used, $x = |L - X|$.

The integral on the right in (11) is the effective interchange and is calculated with the aid of the matrix of partial interchanges $\Delta Sim(T_\xi, P_\xi, T_X, P_X, x)$. In the present case the parameters of the source points T_ξ, P_ξ and the sink T_X, P_X are equally important and must be taken directly at these points (Fig. 4c). It should be borne in mind that the matrix ΔSim is antisymmetric with respect to simultaneous inversion of the temperatures $T_\xi \rightleftarrows T_X$ and the pressures $P \rightleftarrows {}_X$:

$$\Delta\,\mathrm{Sim}(T_i, P_i, T_j, P_j, x) = -\Delta\,\mathrm{Sim}(T_j, P_j, T_i, P_i, x). \tag{15}$$

In the calculated tables we give only half of the matrix ΔSim for $T_\xi > T_X$. Its second half (for $T_\xi < T_X$) can be recorded in a computer memory by using formula (15). Thus, in order to enter into the memory an element of ΔSim when T_ξ = 10,000, T_X = 14,000°K, P_ξ = 1, P_X = 10 bar, we must select from the tables the element for T_ξ = 14,000, T_X = 10,000°K, P_ξ = 10, P_X = 1 bar and write it into the memory with a minus sign. The diagonal of the matrix ΔSim for $T_X = T_\xi$ should be filled with zeros.

After calculating the quantity ∇I, we calculate the divergence of the flux:

$$\nabla S(X) = \int_{(4\pi)} \nabla I(X, \Omega)\,d\Omega. \tag{16}$$

When we use a spherical system of coordinates,

$$\nabla S(X) = \int_0^\pi d\theta \int_0^{2\pi} d\varphi \nabla I(X, \theta, \varphi)\sin\theta. \tag{17}$$

The tables described in the article can be obtained at the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the USSR Academy of Sciences (630090, Novosibirsk, 90, ul. Institutskaya, 4/1). They are prepared in the form of ATsPU printouts, on punched cards, or recorded on the customer's magnetic tape.

NOTATION

I_ν°, spectral intensity of the equilibrium radiation; k_ν', spectral coefficient of absorption; T, absolute temperature; P, pressure; Ω, unit vector; dΩ, element in the space of angular coordinates; x, X, ξ, η, geometric coordinates.

LITERATURE CITED

1. V. G. Sevast'yanenko, "Radiation transfer in a real spectrum. Integration over frequency," Inzh.-Fiz. Zh., 36, No. 2, 218-230 (1979).
2. V. G. Sevast'yanenko, "Radiation transfer in a real spectrum. Integration with respect to the frequency and angles," Inzh.-Fiz. Zh., 38, No. 2, 278-285 (1980).
3. V. G. Sevast'yanenko, "Heat transfer by radiation in the real spectrum," Author's Abstract of Doctoral Dissertation, Physicomathematical Sciences, Novosibirsk (1980).
4. G. A. Koval'skaya, "The effect of internal microfields on the equilibrium properties of a slightly nonideal plasma," Author's Abstract of Doctoral Dissertation, Physicomathematical Sciences, Novosibirsk (1975).
5. G. A. Koval'skaya and V. G. Sevast'yanenko, "Equilibrium properties of low-temperature plasma," in: Properties of Low-Temperature Plasma and Methods of Its Diagnosis [in Russian], Nauka, Novosibirsk (1977), pp. 11-37.
6. V. G. Sevast'yanenko, Effect of the Interaction of Particles in Low-Temperature Plasma on Its Composition and Optical Properties [in Russian], Novosibirsk (1980) (Preprints No. 30 and No. 32, Institute of Theoretical and Applied Mechanics, Siberian Branch Academy of Sciences of the USSR).
7. G. Grim, Broadening of Spectral Lines in Plasma [Russian translation], Mir, Moscow (1978).

8. C. R. Vidal, J. Cooper, and E. W. Smith, "Hydrogen stark-broadening tables," Astrophys. J., Suppl. Ser., 25, No. 214, 37-136 (1973).
9. L. M. Biberman, V. S. Vorob'ev, and G. É. Norman, "Energy radiated by an equilibrium plasma in spectral lines," Opt. Spektrosk., 14, No. 3, 330-335 (1963).
10. R. I. Soloukhin, Yu. A. Yakobi, and A. V. Komin, Optical Characteristics of Hydrogen Plasma [in Russian], Nauka, Novosibirsk (1973).
11. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Academic Press.
12. H. Bethe and E. Salpeter, Quantum Mechanics of Atoms with One and Two Electrons [Russian translation], Gos. Izd-vo Fiz.-Mat. Lit., Moscow (1960).
13. T. Ohmura and H. Ohmura, "Continuous absorption due to free—free transition in hydrogen," Phys. Rev., 121, No. 2, 513-517 (1961).
14. A. Dalgarno and N. F. Lane, "Free—free transitions of electrons in gases," Astrophys. J., 145, No. 2, 623-633 (1966).
15. V. A. Šidlauskas and M. M. Tamonis, "Radiant transfer of energy in a layer of hydrogen plasma. 1. Complete radiation of a hemispherical layer," Tr. Akad. Nauk LitSSR, Ser. B, 4 (101), 81-89 (1977).
16. S. Geltman, "The bound—free absorption coefficient of the hydrogen negative ion," Astrophys. J., 136, No. 3, 935-945 (1962).
17. S. Geltman, "Continuum states of H^- and the free—free absorption coefficient," Astrophys. J., 141, No. 2, 376-394 (1965).
18. I. I. Sobel'man, Introduction to the Theory of Atomic Spectra, Pergamon (1972).
19. R. W. Patch, "Absorption coefficient for hydrogen. II. Calculated pressure-induced H_2—H_2 vibrational absorption in the fundamental region," J. Quant. Spectrosc. Rad. Transfer, 11, No. 9, 1331-1353 (1971).
20. R. W. Patch, "Absorption coefficient for hydrogen. III. Calculated pressure-induced H_2—H vibrational absorption in the fundamental region," J. Quant. Spectrosc. Rad. Transfer, 14, No. 2, 101-110 (1974).

TRANSIENT THERMAL LENS FORMED BY SHORT LASER PULSE IN CONDENSED MEDIUM

É. T. Bruk-Levinson, Z. Plochotski, and I. V. Khodan

UDC 535.42+621.378

The problem of relaxation is solved for a thermal lens which has been formed by a short laser pulse in a condensed medium. An expression is obtained for the focal length of such a lens and the asymptotic trend of the focal length as time increases is determined.

Recent years have witnessed a growing interest in quasioptical structures developing in nonuniformly heated media. Gradients of thermodynamic parameters caused space—time modulation of the refractive index, which alters the optical properties of the medium. It has demonstrated in earlier studies [1, 2] that formation and relaxation of a thermal phase diffraction grating can be successfully used in transient holography as well as for contactless measurements of thermophysical properties of materials. There are also known gaseous lenses used as phase correctors for light guides.

In this communication the results of a study pertaining to a thermal lens formed during passage of a short strong laser pulse through a thin layer of a substance will be presented, whereupon a relation will be established between thermophysical properties of that substance, the space—time variation of the refractive index, and the optical properties of the lens.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Institute of Fundamental Engineering Problems, Polish Academy of Sciences, Warsaw. Translated from Inzhenerno-Fizicheski Zhurnal, Vol. 45, No. 6, pp. 983-987, December, 1983. Original article submitted September 20, 1982.

0022-0841/83/4506- 1429$07.50